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Mark Scheme (Final)

January 2009

GCE

GCE Core Mathematics C3 (6665/01)

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General Marking Guidance

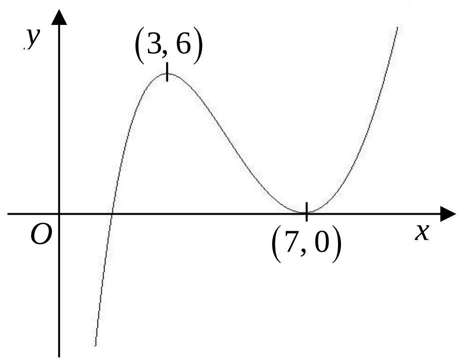
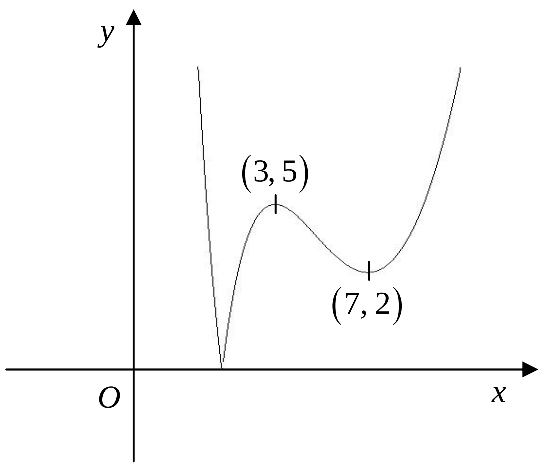
- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

January 2009
6665 Core C3
Mark Scheme

Version for Online Standardisation

Question Number	Scheme	Marks
1.	(a) $\dfrac{d}{dx}\left(\sqrt{(5x-1)}\right)=\dfrac{d}{dx}\left((5x-1)^{\frac{1}{2}}\right)$	
	$=5\times\dfrac{1}{2}(5x-1)^{-\frac{1}{2}}$	M1 A1
	$\dfrac{dy}{dx}=2x\sqrt{(5x-1)}+\dfrac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$	M1 A1ft
	At $x=2$, $\dfrac{dy}{dx}=4\sqrt{9}+\dfrac{10}{\sqrt{9}}=12+\dfrac{10}{3}$	M1
	$=\dfrac{46}{3}$ Accept awrt 15.3	A1 **(6)**
	(b) $\dfrac{d}{dx}\left(\dfrac{\sin 2x}{x^2}\right)=\dfrac{2x^2\cos 2x-2x\sin 2x}{x^4}$	M1 $\dfrac{\text{A1+A1}}{\text{A1}}$
		(4) **[10]**
	Alternative to (b)	
	$\dfrac{d}{dx}\left(\sin 2x\times x^{-2}\right)=2\cos 2x\times x^{-2}+\sin 2x\times(-2)x^{-3}$	M1 A1 + A1
	$=2x^{-2}\cos 2x-2x^{-3}\sin 2x\quad\left(=\dfrac{2\cos 2x}{x^2}-\dfrac{2\sin 2x}{x^3}\right)$	A1 **(4)**

Question Number	Scheme	Marks	
2.	(a) $\dfrac{2x+2}{x^2-2x-3}-\dfrac{x+1}{x-3}=\dfrac{2x+2}{(x-3)(x+1)}-\dfrac{x+1}{x-3}$		
	$=\dfrac{2x+2-(x+1)(x+1)}{(x-3)(x+1)}$	M1 A1	
	$=\dfrac{(x+1)(1-x)}{(x-3)(x+1)}$	M1	
	$=\dfrac{1-x}{x-3}$ Accept $-\dfrac{x-1}{x-3},\ \dfrac{x-1}{3-x}$	A1	**(4)**
	(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1-x}{x-3}\right)=\dfrac{(x-3)(-1)-(1-x)1}{(x-3)^2}$	M1 A1	
	$=\dfrac{-x+3-1+x}{(x-3)^2}=\dfrac{2}{(x-3)^2}$ * cso	A1	**(3)**
			[7]
	Alternative to (a)		
	$\dfrac{2x+2}{x^2-2x-3}=\dfrac{2(x+1)}{(x-3)(x+1)}=\dfrac{2}{x-3}$	M1 A1	
	$\dfrac{2}{x-3}-\dfrac{x+1}{x-3}=\dfrac{2-(x+1)}{x-3}$	M1	
	$=\dfrac{1-x}{x-3}$	A1	**(4)**
	Alternatives to (b)		
	① $\mathrm{f}(x)=\dfrac{1-x}{x-3}=-1-\dfrac{2}{x-3}=-1-2(x-3)^{-1}$		
	$\mathrm{f}'(x)=(-1)(-2)(x-3)^{-2}$	M1 A1	
	$=\dfrac{2}{(x-3)^2}$ * cso	A1	**(3)**
	② $\mathrm{f}(x)=(1-x)(x-3)^{-1}$		
	$\mathrm{f}'(x)=(-1)(x-3)+(1-x)(-1)(x-3)^{-2}$	M1	
	$=-\dfrac{1}{x-3}-\dfrac{1-x}{(x-3)^2}=\dfrac{-(x-3)-(1-x)}{(x-3)^2}$	A1	
	$=\dfrac{2}{(x-3)^2}$ *	A1	**(3)**

Question Number	Scheme		Marks	
3.	(a)			
		Shape $(3, 6)$ $(7, 0)$	B1 B1 B1	 **(3)**
	(b)			
		Shape $(3, 5)$ $(7, 2)$	B1 B1 B1	 **(3)** **[6]**

Question Number	Scheme		Marks
4.	$x = \cos(2y + \pi)$		
	$\frac{\mathrm{d}x}{\mathrm{d}y} = -2\sin(2y + \pi)$		M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2\sin(2y + \pi)}$	Follow through their $\frac{\mathrm{d}x}{\mathrm{d}y}$ before or after substitution	A1ft
	At $y = \frac{\pi}{4}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2\sin\frac{3\pi}{2}} = \frac{1}{2}$		B1
	$y - \frac{\pi}{4} = \frac{1}{2}x$		M1
	$y = \frac{1}{2}x + \frac{\pi}{4}$		A1 **(6)**
			[6]

Question Number	Scheme	Marks
5.	(a) $\quad \mathrm{g}(x) \geq 1$	B1 **(1)**
	(b) $\quad \mathrm{f\,g}(x) = \mathrm{f}\left(\mathrm{e}^{x^2}\right) = 3\mathrm{e}^{x^2} + \ln \mathrm{e}^{x^2}$	M1
	$= x^2 + 3\mathrm{e}^{x^2} \quad *$	A1 **(2)**
	$\left(\mathrm{fg} : x \mapsto x^2 + 3\mathrm{e}^{x^2}\right)$	
	(c) $\quad \mathrm{fg}(x) \geq 3$	B1 **(1)**
	(d) $\quad \dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^2 + 3\mathrm{e}^{x^2}\right) = 2x + 6x\,\mathrm{e}^{x^2}$	M1 A1
	$2x + 6x\,\mathrm{e}^{x^2} = x^2\,\mathrm{e}^{x^2} + 2x$	
	$\mathrm{e}^{x^2}\left(6x - x^2\right) = 0$	M1
	$\mathrm{e}^{x^2} \neq 0, \quad 6x - x^2 = 0$	A1
	$x = 0, 6$	A1 A1 **(6)**
		[10]

Question Number	Scheme		Marks	
6.	(a)(i) $\sin 3\theta = \sin(2\theta + \theta)$			
	$= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$			
	$= 2\sin\theta\cos\theta.\cos\theta + (1 - 2\sin^2\theta)\sin\theta$		M1 A1	
	$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$		M1	
	$= 3\sin\theta - 4\sin^3\theta$ ∗	cso	A1	**(4)**
	(ii) $8\sin^3\theta - 6\sin\theta + 1 = 0$			
	$-2\sin 3\theta + 1 = 0$		M1 A1	
	$\sin 3\theta = \frac{1}{2}$		M1	
	$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$			
	$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$		A1 A1	**(5)**
	(b) $\sin 15° = \sin(60° - 45°) = \sin 60° \cos 45° - \cos 60° \sin 45°$		M1	
	$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$		M1 A1	
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ ∗	cso	A1	**(4)**
				[13]
	Alternatives to (b)			
	① $\sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$		M1	
	$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$		M1 A1	
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ ∗	cso	A1	**(4)**
	② Using $\cos 2\theta = 1 - 2\sin^2\theta$, $\cos 30° = 1 - 2\sin^2 15°$			
	$2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\sqrt{3}}{2}$			
	$\sin^2 15° = \frac{2 - \sqrt{3}}{4}$		M1 A1	
	$\left(\frac{1}{4}(\sqrt{6} - \sqrt{2})\right)^2 = \frac{1}{16}(6 + 2 - 2\sqrt{12}) = \frac{2 - \sqrt{3}}{4}$		M1	
	Hence $\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ ∗	cso	A1	**(4)**

Question Number	Scheme	Marks
7.	(a) $\mathrm{f}'(x) = 3\mathrm{e}^x + 3x\mathrm{e}^x$	M1 A1
	$3\mathrm{e}^x + 3x\mathrm{e}^x = 3\mathrm{e}^x(1+x) = 0$	
	$x = -1$	M1 A1
	$\mathrm{f}(-1) = -3\mathrm{e}^{-1} - 1$	B1 **(5)**
	(b) $x_1 = 0.2596$	B1
	$x_2 = 0.2571$	B1
	$x_3 = 0.2578$	B1 **(3)**
	(c) Choosing $(0.257\,55, 0.257\,65)$ or an appropriate tighter interval.	M1
	$\mathrm{f}(0.257\,55) = -0.000\,379\ ...$	
	$\mathrm{f}(0.257\,65) = 0.000\,109\ \ldots$	A1
	Change of sign (and continuity) $\Rightarrow$ root $\in (0.257\,55, 0.257\,65)$ ∗ cso	A1
	($\Rightarrow x = 0.2576$, is correct to 4 decimal places)	**(3)** **[11]**
	Note: $x = 0.257\,627\,65\ \ldots$ is accurate	

Question Number	Scheme		Marks	
8.	(a) $R^2 = 3^2 + 4^2$		M1	
	$R = 5$		A1	
	$\tan\alpha = \frac{4}{3}$		M1	
	$\alpha = 53\ ...\ ^\circ$	awrt 53°	A1	**(4)**
	(b) Maximum value is 5	ft their R	B1 ft	
	At the maximum, $\cos(\theta - \alpha) = 1$ *or* $\theta - \alpha = 0$		M1	
	$\theta = \alpha = 53\ ...\ ^\circ$	ft their α	A1 ft	**(3)**
	(c) $\mathrm{f}(t) = 10 + 5\cos(15t - \alpha)^\circ$			
	Minimum occurs when $\cos(15t - \alpha)^\circ = -1$		M1	
	The minimum temperature is $(10-5)^\circ = 5^\circ$		A1 ft	**(2)**
	(d) $15t - \alpha = 180$		M1	
	$t = 15.5$	awrt 15.5	M1 A1	**(3)**
				[12]